pears that customary unfavorable low-speed induced drag limitations can be ameliorated by these means.

With regard to flapping frequency, it is clear from (1′) that any value in excess of V/l leads to positive thrust, numerical coefficients of both terms on the right side being positive. At low forward speed, therefore, f may be arbitrarily small, the limit of static thrust ($V = 0$) corresponding to thrust proportional to square of frequency. It is also found that no value of flapping frequency below V/l leads to positive thrust, regardless of speed V.

The contrast with steady-flow force phenomena is further clarified by interpreting separately the two terms on the right side of (1). The first of these, despite the striking formal resemblance to the Kutta-Joukowsky steady lift formula, exhibits a completely different character in the present case. Instead of circulation proportional to speed, as in the steady airfoil theory, Eq. (2) shows that Γ is proportional to frequency in the limit of low speeds, so that the thrust contribution represented by the term in question vanishes only as the first power of V. At higher speeds this term approaches the V-square dependence as in steady flow. The second term, containing a part independent of speed V, therefore dominates the force effects at low speeds. This term has no counterpart in steady lifting flows.

Reference

[1] Pérès, J., *Mécanique des Fluides* (Gauthier-Villars, Paris, 1936), p. 188.

Transcendental Approximation for Laminar Boundary Layers

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In the fields of boundary-layer theory and unsteady thermal conduction, there is considerable use for profile methods.[1,2] These methods are applied to the solution of problems where a function is known to exist, usually in a semi-infinite space, which is known to have its greatest variation in magnitude close to one of the boundaries. Usually it is known that the function is monotonic also. The profile methods that are commonly used at present are approximations in two ways: first the outer boundary condition is brought from infinity to a finite distance from the inner surface, and then the profile is approximated by the use of a polynomial. The profile is then required to satisfy some integral condition. Since the region over which the function is approximated is finite, Weierstrass' theorem can be invoked to protect the use of the polynomial. However, it would seem that this theorem could be too strong for what is required. Furthermore, the degree of closeness of a low-order polynomial to the true function is very small, and this can adversely affect computations of hydrodynamic stability.

There seems to be some merit in exploring the possibility of finding simple, transcendental, approximate profiles that could be used in such integral methods. In order to obtain some hint of a suitable profile, it is useful to examine available numerical solutions for a typical problem, the incompressible boundary-layer flow over a wedge, i.e., the Falkner-Skan-Hartree problem. The exponential function is an elementary transcendent, and since the profiles resemble "decay" curves it is natural to examine the local logarithmic decrement. This is displayed in Fig. 1, in which η is the dimensionless space similarity parameter, f is the dimensionless stream function, and β is the wedge parameter; $\beta > 0$ corresponds to accelerated flows. From the figure it is seen that there is a considerable indication of linearity of the local logarithmic decrement; the profile converges with increasing strength to its outer boundary condition. The linearity is particularly noticeable for $\beta \geq 0$ and remains a reasonable approximation for $\beta < 0$. Accordingly, a profile function can be written

$$\text{pro}\eta = \exp[-\exp(a + b\eta)]/\exp[-\exp a]$$

such that $\text{pro}\eta = 1$ at $\eta = 0$, $\text{pro}\eta \to 0$ as $\eta \to \infty$, and where the function is monotonic, provided that a and b are real and b is positive. For severe cases with $\beta < 0$, it might be possible to use inner and outer expansions of this linear type, or, alternatively, to use an argument $(a + b\eta + c\eta^2)$.

Computations for the wedge-flow laminar boundary layer have been performed using the profile function already given to obtain both velocity profiles and minimum critical Reynolds numbers as functions of the wedge parameter β. The profile function contains two parameters that are functions of β, and, consequently, it is necessary to use two simultaneous

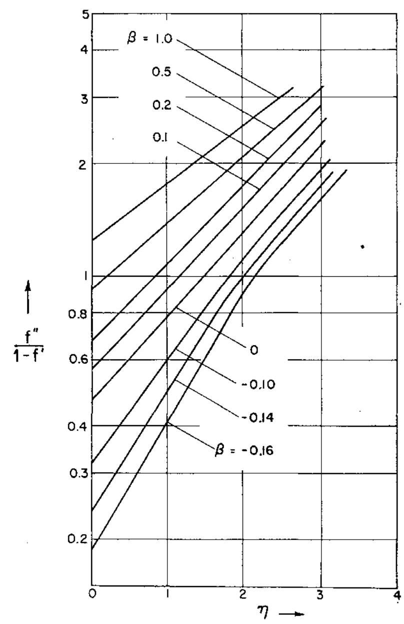

Fig. 1 Local logarithmic decrement in the wedge-flow boundary layer.

equations (the integral momentum and energy equations) to determine them. It was found that the integrals involved could be reduced quite readily to the exponential integral, which is extensively tabulated. Comparison of this profile function with the exact solution for both wall shear and minimum critical Reynolds number is excellent, except for values of β close to that for separation. For values of $\beta > -0.08$, the shear stress is within 0.3% of Hartree's values, and the minimum critical Reynolds number is within 3% of Tetervin's values,[3] which are based on Hartree's profiles. Details of the computations are available elsewhere.[4]

References

[1] Schlichting, H., *Boundary Layer Theory* (McGraw-Hill Book Co. Inc., New York, 1960), 4th ed., Chap. XII.

[2] Lardner, T. J., "Biot's variational principle in heat conduction," AIAA J. 1, 196–206 (1963).

[3] Tetervin, N., "Study of the stability of the incompressible laminar boundary layer on infinite wedges," NACA TN 2976 (August 1953).

[4] Hanson, F. and Richardson, P. D., "A study of the laminar boundary layer equations using a transcendental approximation," Brown Univ., Div. Eng. (August 1963).

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